Navigating a car by a remote steering wheel

View Accompanying Squeak Projects

This project was evoked by, empowered by and is based on the first 15 pages of the book: Powerful Ideas in the Classroom, by BJ Allen-Conn and Kim Rose, 2001.

Introduction

The goal of these Squeak activities is to use mathematics to improve the design of a remote (wheel) control car. Through these activities we give mathematical concepts concrete meaning and purpose.[1]

The math concepts and terminology addressed in these activities include:
X and Y coordinates, graphs, functions, domain and range, rates of change, discontinuity, scaling and derivative. Any, some, or all of these can be discussed.

Instructions and Discussion

Create (by painting): a car and a steering wheel (As 2 different objects).
Use the car script: "Forward by 5" (or any speed you feel comfortable driving safely) as an "engine"
Change car's directions by rotating a steering wheel (using the blue rotating handle in its halo) and transmitting its headings to the car. The transmission can be done in two ways [2]:

A. **Scripting the car to get its heading from** (align itself with) **the wheel's heading.**
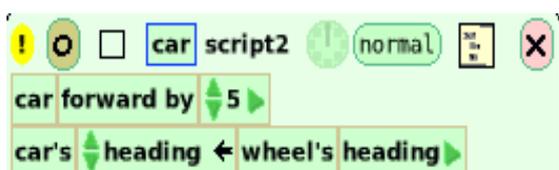
B. **Scripting the car to get the turning measure** from the wheel's heading.
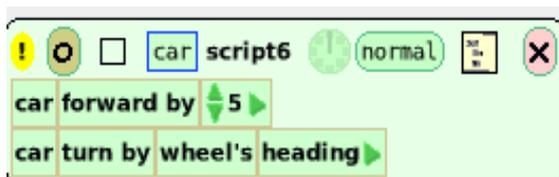
The goal: To make navigating the car by the wheel as easy and accurate as possible. **In particular to find ways to make the car less sensitive to the wheel's movements so that small movements of the wheel will not yield a catastrophic consequence for the car.**

Here are some suggestions that can be explored by fifth graders and up, either as a class activity, or in small groups. Each group can explore all the possibilities, or only part of them. The graphing is not recommended before algebra I.

Using only script A

A1. Use a bigger wheel.

Results: This way the arc measure for each degree is longer and thus same hand movements can yield smaller change in wheel's headings. The car is not less sensitive to wheel's movement; the wheel's heading is less sensitive to hand's movements. In any case it is easier to get closer to the desired turning degree measure.

A2. Many times we use subtraction to reduce the value of a number. Try to reduce the value of "wheel's heading" by subtracting a certain amount from it, let's say 20 degrees.

Does it make the car turn less sharply than the wheel? Does it make driving easier than without it?

Results: There is a difference between the car's heading and the wheel's heading, but both change the same number of degrees, in the direction of the wheel movement. (When the wheel changes heading 35 degrees, so does the car.) The accuracy of the car's direction is still as good, or bad, as our hands' movements.

A3. Reduce the movement of the car by using a fraction of "wheel's heading" instead of its entire value. For example:

Results: The car no longer keeps the same heading as the wheel, and does not turn as sharply as the wheel. In the example, wheel's heading is divided by 2 and the car turns half the turn of the wheel. This way the car movement is less sensitive to the turning of the wheel. ***Good! But****, the car's heading angle (h) is limited to: 180/2 ≥ h ≥ -180/2. In other words when we divided by 2, the car's heading can not get values greater than 90 degrees or smaller than negative 90 degrees. The reason for this is: the values given to headings are growing from 0 to 180 as we turn clockwise and decreasing from 0 to minus 180 as we turn*

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Discontinuity: the car's heading does not behave nicely (in continuity) whenever the heading of the wheel passes the 180 degree mark. When the wheel's heading is 180, the car heading will be 180/2 or 90. When we move the wheel a little further clockwise, say heading -178 degrees, the car's heading will be -178/2=-89, almost the completely opposite direction. This idea of discontinuity is an important concept in mathematics.

Using only Script B
Without any measures to lower its rate, the car quickly spins out of control
List of measures that can be tried in order to make "car turn" by wheel's heading more controllable:

B1. Reduce the value by which the car is turning by subtracting something from the wheel's headings. For example reduce the heading by 15.
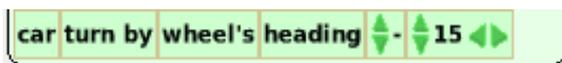
Results: It is a very confusing experience. We expect the car to go straight when the wheel heading is zero, but in this case it will turn by -15 whenever the car's heading is zero. This action does lower the value of "turn by" but we are interested in the absolute value, and subtraction will reduce it only for values that are positive. For negative values of heading, the car turns even sharper than without the subtraction!
In addition, whenever the value of the wheel's heading is between zero and 15 we expect the car to turn clockwise, but the turn values are going to be negative and, therefore, the car will turn counterclockwise. The same problem of changing direction will appear when the wheel's heading is smaller then -165 degrees.

B2. Take a fraction of the "wheel's heading" instead of the full value.
For example divide by 10.

Results: Using the wheel gets to be more like using a real life car's steering wheel. Small moves clockwise and small moves counter clockwise will yield the whole range of car headings. The car continues to turn as long as the heading of the wheel is not 0. The effect

*of a small movement of the wheel can be multiplied by the repeated TURN executions. In order to keep movement under control one needs to take only a **small** fraction of the value of wheel's heading (try one tenth) and move the wheel back to heading zero as soon as car gets close to the desired direction (but **before** it get there!) Keep in mind that moving the wheel **back** to zero **will not change** the direction the car is turning in! The car and the wheel no longer keep the same heading and they don't always even **turn in the same direction!** The wheel can move counterclockwise and car will continue to turn clockwise! That is true for real life cars turning as well.*

The range of values the "turn by" can get from the wheel's heading is limited (-180/A<=h<=180/A) but that does not limit the car headings, we might need repeated executions of "turn by" to get to the right heading for the car, but we can achieve any heading we wish, with a small range of values for the"car turn by" script.
 *The heading of 180 for the wheel is still a **discontinuity** point for the turn values, but there is no need to get close to it. In fact we have to actively stay away from it! (It does not limit the movement of the car!)*

Speaking (Squeaking) Mathematics

Coordinate Plane
For the very young, making a button for reset is a great exercise in the meaning of x,y and heading values. Such a script is very useful for bringing the car back if it moves off the screen or disappears under another object, and to position it for restarting.

Directions for RESET BUTTON: Drag a "press me" button from Supplies and open its viewer. Put the car and the wheel in the place you choose for reset. Take an empty script and drag the values you need from the car and wheel viewers.

IMPORTANT: be sure to make the script work only when **mouse down (**by choosing "mouse down" from menu on top, near the clock).

An example:

A nice addition for using coordinates could be a set of obstacles around which the drivers will practice their driving. A set of class-wide obstacles (all positioned at the same coordinates and of the same size) will let them compare more objectively the quality of their scripts and use some of the Geometry menu features.
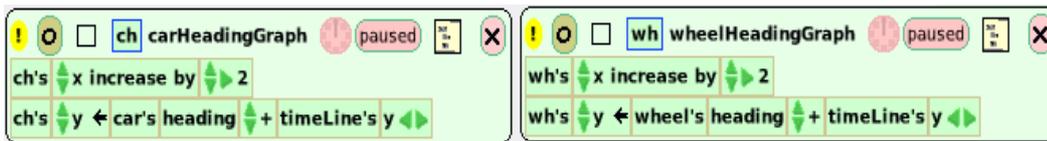

GRAPHING
Making scripts for graphs, representing the heading of the wheel and the heading of the car at any given moment, is a way to translate the concept of graphing into its basic meaning; Representing a set of numbers by a set of visual objects to illustrate a chosen property of the original set. Here we represent each value of heading by a distance of a point from a line. Points that are further up from the line represent higher numeric values of headings.

Directions for the graphing tool: paint 3 objects: A line for use as "time-line". Two small round buttons, ("wh" red for the red wheel's headings, and "ch" green for the green car's headings). Script these buttons to get their y value from the heading of the car and the wheel respectively at any given time, and their x value to increased by one (can be 2), at each clock's tick. The trail they leave by using "pen down" is a graph of the headings.
 In order to use the "line" as a time-line, make sure the buttons are on the left side of the line at the beginning of the activity (reset button) and adjust their y values to that of the line by adding the y value of the line to their y values respectably. (In the accompanied squeaklet the "Timeline" rotating center was moved to its left end, so its XY coordinate are those of its left end).
See example's scripts below.

In this activity, **graphing clearly provides additional information:**
1) The ability to encapsulate the history of movements, and thus to explore the connection between them.
2) The encapsulated history lets us see, and to most of us it is **very new**, that the car does not always follow the wheel's direction of turning. (See the discussion above or in the activity book)

3) The graph illustrates, to the mostly surprised students, the fact that the headings' values are between 180 degrees and -180 degrees (and not 0 to 360 as most of them expected.). This is a choice the creators of Squeak made. Due to this choice the point of discontinuity is at 180 degrees and not at zero. That makes it easier to use gears for navigating a car. It is a great demonstration of differences between mathematics as an abstract field and mathematics as tool for modeling. Two mathematically equivalent systems are not always the same.

Warning: It takes experience to read the graphs. It is harder to analyze them when car gets its turn measure from the wheel. Start with graphic interpretation of very simple movements like:
1. Wheel heading is 0, car driving on a straight line; here car heading is a line, and wheel heading is the zero line.
2. Wheel heading is a small positive number. Here car is turning in a constant rate thus drawing a circle. Wheel's heading graph is a horizontal line, and car's heading consists of slanted segments with the same positive slope, ranging between -180 and +180 degrees.
3. Car doing only one left turn.

GEAR
Scaling down: Check and compare properties of subtraction and division in this context. (Subtraction lowers the "size", changes the ratio, but keeps differences and order intact. Division by a positive number, does not keep "size" and differences, but keeps other properties of numbers like order and ratio.
This subject can be used in the context of Mathematical Modeling (model for the gear of a car or scaling for geographical maps) and in mechanics of "real objects" (Car's gear or Analog clocks).

Possible expansions of the subject of gears are:
Check mechanical gears on one hand, and uses of "gears" in the computer on the other hand. (E.g.: Explore plastic models of gears, models of pendulum clocks or any other "real life"

gears. Adjusting sensitivity of the "mouse" in computer is probably an example of a built-in "software gear". Making a clock in Squeak is another.)

DOMAIN AND RANGE OF A FUNCTION:

The car's heading gets its values from the wheel's headings through number of functions. When the car gets heading values from the wheel (A1,A2, A3) the domain is full range of headings; between -180d degrees to +180 degrees. The range for A1 and A2 is the same as its domain. The range in A3 is between -180/n degrees to +180/n degrees (n is the number by which we divide the wheels heading, the gear).

When car gets turn measure from wheel's heading, the domain is smaller: One usually uses small portion of wheel's headings (don't turn the wheel more then 90 degrees, and most of the time much less) and yet get the range is the full range of headings.

CALCULUS:

The case where the wheel's headings are used as car turn's measure is a nice demonstration of two related functions. A function $F(t)$ = the car's heading at any given time, and another function $f(t)$ = the wheel's heading at any given time.

The wheel's headings give the car the values by which to turn which are **the rates of change** for the car's heading at any given moment. In Math terminology: **The wheel's heading is the DERIVATIVE of the car's headings.**

We can see: (among other things)

- Whenever wheel's heading is 0, car moves in straight lines. Derivative zero indicates no change of direction.
- Whenever wheels heading is positive car turns clockwise. We can turn the wheel right or left, as long as its heading is positive the car will keep turning clockwise. That demonstrates that a positive derivative at an interval indicates the function's values (car's heading values) are increasing.
- Whenever wheel's heading is negative the car will turn counter- clock direction, regardless of the direction of the move of the wheel. That demonstrates that negative derivative at an interval indicates the function's values (car's headings values) are declining at that interval.

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1. Our understanding of the Squeak's car movement and its control helps us to improve the design and enhances our understanding of real cars' navigating systems.

2. Both of these methods produce results that affect the car's movement and at the same time give opportunity to use explore and enhance the relevant mathematics.

Related Squeak Document:

Math Navigating a Car by a Wheel: User navigates a car is by a wheel using scaled down wheel's heading for car's turn measure. Graphs of the objects' headings are plotted as well. Activity book gives directions for navigation and the reading of the graphs.

Math Navigating a Car 4 Ways

The 4 alternative navigation's scripts described in the above paper are in the car's viewer, and there is an option for graphs for the car and wheel headings. Warning: When you don't want a script to run, put it in a "normal" mode, otherwise it will run when you click on the "go" button.

Grade Level: Elementary School, Middle School, High School